A Survey of Fuzzy Approaches in Spatial Data Science

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***Abstract*—*Spatial data science* emerges as an important subclass of *data science* and focuses on extracting meaningful information and knowledge from *spatial data* to enable effective communication and interpretation of both spatial data and analytic results. It emphasizes the importance of location and spatial interaction by storing, analyzing, retrieving, and visualizing spatial and geometric information. Frequently, spatial objects are afflicted by *spatial fuzziness*, characterizing spatial objects with blurred interiors, uncertain boundaries, and imprecise locations. Fuzzy set theory and fuzzy logic have become powerful tools to adequately represent spatial fuzziness. This paper provides a survey and a review of the literature to understand the application of fuzzy approaches to spatial data science (projects) with the objective of proposing, motivating, and envisioning *fuzzy spatial data science*.**

***Index Terms*—Spatial data science, fuzzy spatial data, spatial fuzziness, fuzzy spatial reasoning**

I. Introduction

Data science is the study of *data*. It is a data-driven and highly interdisciplinary field (see Figure 1). It focuses on designing methods for capturing (data acquisition, data extraction), representing (data modeling, data structures), maintaining (databases, data warehouses, data cleaning), processing (data mining, machine learning, data clustering, data classification, data summarization), and analyzing (statistics, exploratory analysis, confirmatory analysis, regression, qualitative analysis, business intelligence) large amounts of alphanumerical data (big data). It pursues the objectives to effectively extract useful information, gain insights and knowledge from any type of data (structured, semi-structured, unstructured), and enable effective communication and interpretation of both data and analytic results (data reporting, data visualization, decision making, prediction, recommendation, pattern detection, anomaly detection, optimization, scoring, ranking). Similar definitions of data science are discussed in [1], [2].

Spatial data science is a subclass of data science in an object-oriented sense. It inherits all concepts and methods from data science but widens the scope of alphanumerical data to *spatial data*, adds special concepts and methods for exploring spatial data, and particularly considers their special geometric and topological characteristics such as location, distance, and spatial interaction [3], [4]. There is a consensus that special, vector-based data types called *spatial data types* [5] such

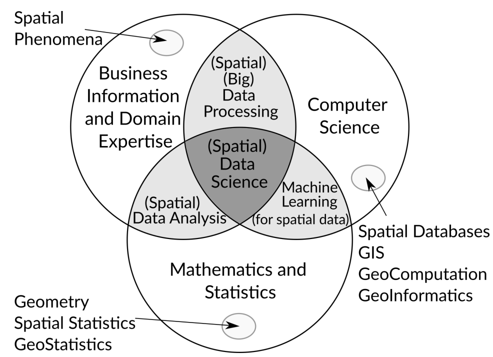

Fig. 1. Venn diagram that shows the interdisciplinarity of data science with respect to three main disciplines represented by the three large circles. A small circle inside each main discipline represents the sub-classes responsible for adequately manipulating spatial data in the corresponding discipline (some of them are listed in the figure). Consequently, spatial data science is an extension of data science by especially focusing on the interaction of these sub-classes.

as *point*, *line*, and *region* in two-dimensional space as well as *surface* and *volume* in three-dimensional space should be deployed in spatial systems to represent and process spatial data. *Spatial objects* are values of these data types and are processed quite differently than alphanumerical data. Further, the assumption is that spatial objects are precisely determined entities with exact and well-known locations, shapes, and boundaries. They are hence called *crisp spatial objects* [5]. For instance, point objects may represent the exact locations of power towers, line objects may constitute the exact course of streets, and region objects may comprise the exact boundary of states. Further, the information extracted from them is also exact. For instance, topological relationships on spatial objects are well-defined, the area measure of a region is crisp, and the distance between clusters of points in space is accurate.

However, many spatial phenomena are characterized by *spatial fuzziness* (*spatial vagueness*) (e.g., [6]–[11]). This feature captures the inherent property of many spatial objects in reality that have inexact locations, vague boundaries, and/or blurred interiors, and hence cannot be adequately represented by crisp spatial objects. A spatial object is fuzzy and thus called *fuzzy spatial object* if it contains locations that cannot be assigned completely to the object or to its complement.

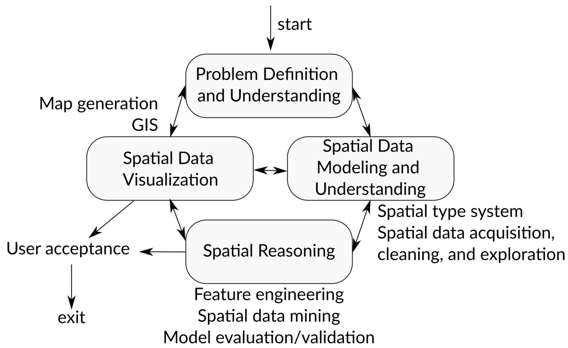

Fig. 2. Proposed life cycle model of a spatial data science project.

Examples are air polluted areas, temperature zones, soil strata, agricultural cultivation areas, and habitats of species. *Fuzzy set theory* and *fuzzy logic* [12] have become popular formal tools among scientists to deal with spatial fuzziness and to model and process fuzzy spatial objects. Integrating them into spatial data science would lead to *fuzzy spatial data science*.

This survey paper has four main goals. The first goal is to identify and comprehend the components of projects that apply methods of spatial data science when designing applications, called *spatial data science projects* in the following. For this, we present a life cycle model for spatial data science projects by extending the life cycle models of conventional data science projects (e.g., [2], [13]). Our life cycle model shows how its components interact and deal with spatial data. The second goal is to motivate the inclusion of the aspect of fuzziness into spatial data science. The third goal is to extend these components to deal with spatial fuzziness. Three main aspects are discussed in this context: (i) understanding and data modeling of spatial fuzziness, (ii) the spatial reasoning process with crisp and fuzzy spatial data, and (iii) visual analysis of crisp and fuzzy spatial objects. The fourth goal is to discuss challenges and limitations of current approaches and identify open research opportunities in fuzzy spatial data science.

This paper is organized as follows. Section II presents a life cycle model of a spatial data science project. Sections III, IV, and V discuss fuzzy approaches to its components. Section VI explores challenges and limitations. Finally, Section VII concludes the paper and presents future work.

II. Life Cycle of a Spatial Data Science Project

This paper focuses on a literature review of a set of fuzzy approaches that do or might contribute to the components of a spatial data science project. Figure 2 depicts our proposed life cycle model of a spatial data science project. It extends the life cycle models of conventional data science projects (e.g., [2], [13]) to deal with spatial data (handling). An essential goal of a spatial data science project is to integrate the four main components of the life cycle model to achieve user acceptance.

The first component and step of the life cycle model is problem definition and understanding. This is a well-known part of the scientific method and includes the formulation of hypotheses that serve as a foundation for solving a given problem. It guides us to leverage data models and methods and provides us with guidelines to evaluate and analyze the obtained results. In a spatial data science project, this component seeks to understand what is being represented by spatial phenomena and to find out if the phenomena vary in time or/and are characterized by spatial fuzziness. The interaction with the components that model, store, and visualize spatial data aids in and refines (as needed) the identification and characterization of spatial phenomena.

The second component is spatial data modeling and understanding. It refers to adequately acquiring, cleaning, and exploring spatial phenomena that are formally defined by spatial type systems, represented by well-defined data structures, and stored as spatial objects in spatial databases. As pointed out in Section I, spatial objects can be characterized by spatial fuzziness and lead to fuzzy spatial objects. Section III discusses fuzzy approaches that can be applied to this life cycle component. The interaction of this component with other components permits us to visualize spatial objects and apply them to spatial reasoning engines.

The third component is spatial reasoning. It consists of models for mining and extracting meaningful information on spatial datasets. Two different approaches can be employed. The first approach relates to the analysis of spatial data without using alphanumerical attributes. An example is the application of a clustering algorithm to create groups of nearest points. The second approach refers to the analysis of spatial data annotated by alphanumerical attributes. In this case, the approaches additionally characterize spatial objects by considering thematic values associated to each point in space. In both kinds of analyses, we can perform predictions, recommendations, and estimations. Section IV details several fuzzy approaches that can be applied for these purposes. The meaningful information and insights generated from this component can be visualized or stored for further analysis.

The last component is spatial data visualization. This component employs map generation techniques and Geographical Information Systems (GIS) to better understand the nature of the real-world phenomena represented by spatial objects, identify possible spatial relations, and interpret the results of spatial reasoning models. Section V discusses how fuzzy approaches impact on the visualization process.

III. Fuzzy Approaches to Spatial Data Modeling and Understanding

In this section, we discuss approaches that apply fuzzy set theory and fuzzy logic to the elements encompassed by spatial data modeling and understanding. These elements are (i) spatial data modeling, (ii) spatial data acquisition, (iii) spatial data cleaning, and (iv) spatial data exploration (see Figure 2).

Spatial data modeling defines the concepts and implementation methods needed to represent, store, and retrieve spatial phenomena in applications. Fuzzy set theory can be employed to adequately represent spatial fuzziness. This has led to *fuzzy spatial data types* for fuzzy points, fuzzy lines, and

(a) (b) (c)

Fig. 3. A fuzzy point object (a), a fuzzy line object (b), and a fuzzy region object (c). Darker areas indicate higher membership degrees than lighter areas.

fuzzy regions (Figure 3). A fuzzy spatial object assigns a membership degree between 0 and 1 to each of its points in order to indicate the degree to which a point belongs to the object. An example is a fuzzy region object that represents the habitat of a species and is based on a membership function that models the presence of a species in a given point by means of collecting data from remote sensing devices [14].

Several studies (e.g., [8], [15], [16]) focus on the definition of fuzzy regions, i.e., areal vague objects, in applications. Other works (e.g. [7], [9], [17]) provide a spatial type system that also includes fuzzy points and fuzzy lines together with respective operations on them, such as fuzzy geometric set operations (e.g., union, intersection), fuzzy topological relationships (e.g., contains, overlap), and fuzzy numerical operations (e.g., area of a fuzzy region). From an implementation perspective, there are some efforts in the literature that propose *abstract data types* (*ADTs*) for hiding the internals of fuzzy spatial objects and their operations. The authors in [18] introduce the ADT *FuzzyGeometry* that can store fuzzy point objects and fuzzy line objects as attribute values in relational tables of PostgreSQL. The authors in [19] provide ADTs for fuzzy points, fuzzy lines, and fuzzy regions based on the conceptual model in [7] as an extension of the GIS GRASS. The Spatial Plateau Algebra [11] is a specification of an implementation for [9] and offers expressive fuzzy spatial data types and a broad collection of fuzzy spatial operations and predicates. In addition, spatial data is frequently associated with time, leading to *spatiotemporal objects* [20]. This concept has been also applied to fuzzy spatial objects with the specification of *fuzzy spatiotemporal objects* [21].

Spatial data acquisition is essential to start a spatial data science project. It is responsible for capturing spatial phenomena and adequately representing them as crisp or fuzzy spatial objects. There are many spatial capturing methods and publicly available repositories for crisp spatial objects (e.g., volunteered geographic information systems [22]). However, there is a lack of spatial data acquisition methods that consider spatial fuzziness. Frequently, spatial fuzziness of real-world phenomena is obtained by pre-processing crisp spatial objects (e.g., regions representing soil strata) associated with domain-specific information (e.g., each point of a region indicates the soil texture). In [23], the authors employ the Fuzzy C-means (FCM) [24] to fuzzify spatial objects for the delineation of management zones in precision agriculture. A generic approach for building and storing fuzzy region objects has been proposed in [25]. Figure 4 depicts three fuzzy region objects

(a) (b) (c)

Fig. 4. Representing hot (a), warm (b), and cold (c) temperatures in Brazil by using fuzzy region objects.

built using this approach. It shows the average temperatures for the years 1970 to 2000 in Brazil that are subdivided into three classes. Such representations help one understand how much a given point belongs to each class. The same point can have multiple membership degrees in different classes.

Spatial data cleaning is mainly related to the execution of needed adjustments in crisp and fuzzy spatial objects. An example is the deletion of points that potentially are outliers in inference models (Section IV). The concept of spatial data cleaning is also related to spatial data quality and combines concepts of spatial data modeling and spatial data acquisition. Fuzzy regularization is applied to fuzzy spatial objects to remove geometric anomalies (isolated discontinuities) such as punctures and dangling points of fuzzy region objects [7], [9].

Spatial data exploration refers to an approach to analyzing spatial phenomena by summarizing and comprehending the characteristics of spatial objects (e.g., spatial relationships). Issuing fuzzy spatial queries on spatial database systems and GIS is a possible strategy. Hence, it is important that the (fuzzy) spatial type system offers a broad collection of spatial operations. The definition of topological relationships that consider spatial fuzziness has been the focus of many studies. Membership-degree [9] and coverage-degree [26] topological predicates offer different perspectives on topological relationships for fuzzy regions. Fuzzy region connection calculus [27] is an approach to evaluating topological relationships on fuzzy regions to process fuzzy spatial skyline queries. The result of these spatial operations can be used as input in mathematical and statistical models (see Section IV).

IV. Fuzzy Approaches to Spatial Reasoning

In this section, we review fuzzy approaches applied to the elements in the context of spatial reasoning. These elements are (i) feature engineering, (ii) spatial data mining, and (iii) model evaluation/validation (see Figure 2).

Feature engineering aims at selecting the most important attributes and characteristics that describe spatial phenomena to be employed in spatial data mining models. Geographical coordinates are not modified or adjusted in this step. The main issue is to deal with a large number of attributes (i.e., high-dimensional data) extracted from spatial data and thus, face problems like anomalies and the "curse of dimensionality". Several fuzzy strategies have been proposed to mitigate such problems. For instance, in [28], a singular value decomposition is used as a basis for the reduction of high-dimensional data in fuzzy models. Fuzzy genetic algorithms can be also applied to the feature selection problem, as discussed in [29]. Another approach applies three main steps to select features that consist of combining generated fuzzy sets from the dataset and then make a feature ranking based on defuzzification [30]. Specific problems have also been the focus of feature engineering, such as the projection of features in the fuzzy space and then select them based on the consistency measures for handling *classification problems* (see below) [31].

Spatial data mining has been widely studied in the literature and its goals include obtaining new knowledge and discovering spatial patterns. We discuss fuzzy approaches belonging to the following three types of spatial data mining models: (i) spatial inference systems, (ii) unsupervised learning of spatial patterns, and (iii) supervised learning of spatial patterns. Many fuzzy approaches for spatial inference systems (e.g., [6], [32]) extract properties of spatial objects (e.g., the area of a fuzzy region object) and consider them as input of a specific fuzzy inference method (e.g., Mamdani), which yields new conclusions based on fuzzy logic and previous knowledge. Other approaches (e.g., [33], [34]) make use of the membership degrees of points as input of the fuzzy inference method. FIFUS [10] extends and generalizes conventional inference systems by allowing that fuzzy spatial objects are employed as inputs of fuzzy inference methods and that the results are converted into other fuzzy spatial objects.

Unsupervised learning models on spatial data are useful to spatial cluster analysis, which discovers previously unknown patterns and knowledge in spatial datasets without pre-existing labels. FCM is a popular approach used for discovering patterns in point datasets annotated by domain-specific values (e.g., [23], [35]). FCM has also been augmented to deal with fuzzy spatiotemporal data [36]. In [37], an approach based on the Fuzzy Partitioning Around Medoids algorithm and Dynamic Time Warping dissimilarity measure is proposed to cluster spatiotemporal data. Other approaches apply fuzzy neural networks to unsupervised learning models on spatial data. Examples are a neuro-fuzzy approach to recognize classes of land surfaces [38] and a fuzzy deep-learning approach for predicting citywide traffic flow using spatiotemporal data [39].

Supervised learning models on spatial data aim at inferring a (spatial) entity as output for a given a set of (spatial) entities as input by using previous knowledge from training examples. That is, given a set of values, they recognize or approximate another set of output values. Classification algorithms have been adapted to assign membership degrees to each point in space to represent the nature of spatial fuzziness [40]. Spatial variability also focuses on the fuzzy spatial classification or regression, such as its use to determine the concentrations of heavy metals in rivers [41]. Another application relates on extracting crisp-fuzzy spatial association rules by discovering spatial correlations [42]. Fuzzy neural networks have been also used in classification problems with spatial data. Examples include the decision support system for classifying industrial sites [43] and the use of a neuro-fuzzy classifier for improving the accuracy of classification of urban environments [44].

Model evaluation and validation is the final step that extracts statistical measures of built models (as discussed before) to analyze their quality. Examples of measures are root mean square error, mean absolute error, and mean absolute percentage error. They can be captured and used by validation models, such as the holdout and k-fold cross validation [45].

V. Fuzzy Approaches to Spatial Data Visualization

In this section, we discuss fuzzy approaches that propose means and methods to deliver valuable insights from a spatial data science project by using maps and GIS (see Figure 2). The visualization of spatial objects in modern and advanced applications plays an important role in enhancing user experience and achieving user acceptance. Many visualization methods for conventional data science and fuzzy applications have been proposed in the literature (e.g, [46]). However, visualization of spatial fuzziness (uncertainty) in a spatial data science project is challenging [47]. It goes beyond the scope of classical analytical behaviors because of the necessity of interpreting latent spatial information afflicted with spatial fuzziness (e.g., fuzzy relationships, fuzzy boundaries). Further, it should enable the interpretation of recommendations, predictions, and inferences performed by spatial data mining models.

The main goal is to provide visualization methods for representing intrinsic spatial fuzziness, such as fuzzy boundaries and blurred interiors. Some studies have proposed domain-specific visualizations. In [48], the authors propose an uncertainty framework for climate information by introducing a typology that distinguishes different uncertainties, such as epistemic, natural stochastic, and human reflexive. Another example is the visualization of hurricane forecasts by considering their intrinsic uncertainties when analyzing the influence of storm characteristics [49]. In such a study, participants, who were non-experts in the domain, have analyzed five forms of representing a hurricane forecast.

Another goal is to visually understand the meaning of the membership degrees assigned to the points of a fuzzy spatial object [47]. This is particularly useful when interpreting results after processing spatial data mining models (Section IV). A general approach for this purpose is proposed in [50], which provides the visualization of different types of uncertainty that can represent distinct spatial characteristics like the uncertain classification and data accuracy. The effect of spatial fuzziness in decision-making applications has been discussed in [51].

In addition, the underlying approach to modeling a fuzzy spatial object turns out to be relevant in spatial data visualization (Section III). In applications where spatial fuzziness is ignored or misunderstood by non-experts, a fuzzy spatial object is simply viewed as a classical crisp region object without any correlation with membership degrees (Figure 5a). Some approaches (e.g., [52]) are based on a three-valued logic with the truth values *true* (all points with membership degree 1), *false* (all points with membership degree 0), and

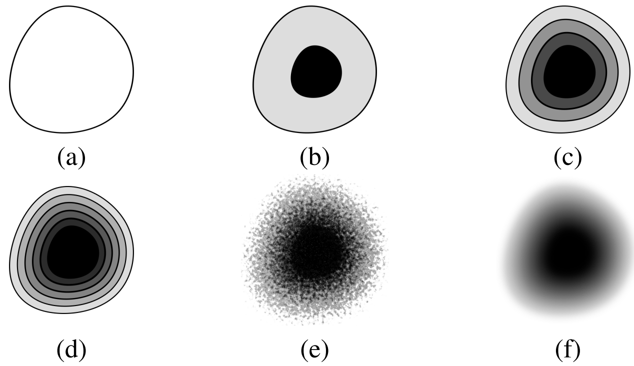

Fig. 5. Different visualizations of a fuzzy region object: negligence of its fuzzy boundary (a), representation by using three-valued logic (b), application of 4 and 6 levels of spatial fuzziness (c) and (d), use of random dots for representing its fuzzy boundary (e), and use of a dense representation for the smooth transition among the membership degrees (f).

maybe (all points in the interval $]0, 1[$). However, they limit the expressiveness of spatial fuzziness (Figure 5b). The Spatial Plateau Algebra [11] considers a fuzzy region object as a finite collection of disjoint components, where each of them represents one level of spatial fuzziness (Figures 5d and 5e). Some other approaches (e.g. [7], [9], [53]) allow the assignment of any value of $[0, 1]$ to each point of a fuzzy region. By employing the visualization method in [54], we can use a color scale for viewing fuzzy boundaries in a *visual equality*. The employed color to represent a point equally shows to which extent this point belongs to the phenomena. Another aspect is the density of the points when visualizing a fuzzy spatial object. For instance, the use of random points to denote a fuzzy boundary can introduce more uncertainty to given phenomena [54] (compare Figures 5e and 5f).

VI. Challenges and Limitations

Many of the discussed approaches face some limitations that impact on the applicability of fuzzy spatial data science projects. These limitations open new research opportunities that are summarized as follows.

a) Abstract data types for fuzzy spatial data: Extensible database systems and GIS permit us to specify and implement ADTs for complex objects in a unique data structure. The main advantage of an ADT is that users can manipulate complex objects without knowing their underlying complex structure and algorithms. Unfortunately, current approaches have different shortcomings when proposing ADTs for fuzzy spatial data, such as the lack of a broad collection of fuzzy spatial operations (see [11] for more details). A consequence of this problem is, to the authors' knowledge, that the natural capturing and storing of data afflicted with spatial fuzziness does not exist in practice. Currently, users have to associate collected crisp spatial objects with domain-specific alphanumeric data to produce the notion of spatial fuzziness. Hence, spatial uncertainty should be taken into account when capturing fuzzy spatial data, such as in crowd-sourcing applications (see [47]).

b) Optimizing methods for dealing with big fuzzy spatial data: The volume of information affects the method how data should be manipulated. Spatial fuzziness even complicates the situation. Some initial works in the literature have focused on this problem for manipulating large fuzzy spatial datasets (e.g., [16]). On the other hand, distributed and parallel systems are powerful tools that have not been explored in this context. Hence, optimizations based on spatial Hadoop-based systems and Spark-based systems [55] should be addressed since uncertainty is a feature of an increasing number of applications.

c) Integration of the components in fuzzy spatial data science projects: A fuzzy spatial data science project consists of unifying a set of complex components. In conventional data science projects, there are some integrated environments, such as the Apache MADlib [56] and Orange [57]. They help users to make workflows of data structures and algorithms from the data collection to the visual analysis. Such environments offer some kind of support for crisp spatial data but not for fuzzy spatial data. Hence, the specification or extension of such environments for fuzzy spatial science projects is required to decrease the complexity of such projects.

VII. Conclusions and Future Work

In this paper, we have presented a life cycle model of a spatial data science project. We have shown how its components can be extended by fuzzy approaches by providing a comprehensive survey. This shows the importance of adequately representing and manipulating the intrinsic fuzzy nature of spatial objects in the different components of spatial data science projects. In fact, this permits us to identify an emerging, motivating, and envisioning sub-class named *fuzzy spatial data science*. Further, we were also able to discover limitations and discuss new research topics in this field.

Future work aims at developing solutions and strategies to (i) implement libraries and capturing methods for fuzzy spatial data, (ii) optimize fuzzy spatial operations on large spatial datasets, and (iii) design an integrated and generic environment where users can deploy specialized methods and algorithms to deal with fuzzy spatial objects.

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